

# **Degressively proportional division of indivisible goods**

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## **Abstract:**

The article presents the formal definition of degressive proportionality, and also identifies other possible interpretations of this concept. In the next section the article presents submitted proposals for the allocation of mandates, taking into account the additional conditions set forth in the Treaty. These conditions concern the minimum and maximum number of representatives of individual Member countries. There will also be proposed a method of evaluating disproportionality of distribution that makes use of, known in economics of prosperity, the term of Lorenz curve and the Gini coefficient. This allows you to locate the specific divisions among the classical arrangements, namely the equal and proportionate division. The article mentions also the problem of demographic stability of the solutions proposed by, inter alia, the members of the Commission.

## **Key words:**

Fair division, proportional division, degressive proportionality, the European Parliament, the Lorenz curve, Gini coefficient.

## **Introduction**

Formed by centuries the rules of social justice determine specific methods of distribution of wealth, benefits and burdens. The most equitable distribution, according to the rules, shall be the one in which everyone gets an equal share of goods and yet the same proportion of bearing the burdens. Regarding problems associated with formation of electoral laws for collective bodies, the principle of equal division is expressed in the postulate that every deputy, MP or simply a person selected in the elections represented the same number of voters regardless of the constituency from which they come. In view of the frequent inequality of constituencies, this leads to the formation of the so-called proportional allocation rules. According to them, delegates are elected in proportion to the population of individual districts. Therefore, what in terms of constituencies is a proportional division, in relation to all voters becomes an equal division.

Precursors of the development of the theory of proportional divisions were known personalities of the American political scene. To this day, in different electoral laws there exist proposals put forward by Hamilton, Webster, Jefferson and Adams. Postulated by them, the algorithms for allocating seats to the House of Representatives - the lower house of Congress - at the beginning of last century, lived to see their mathematical formalization and were formed as part of a larger theory. Although it is easy to identify weak points suggested by these divisions, there are currently no practical alternatives. Paradoxes that appear in some cases when these divisions are used do not affect the popularity of the methods applied for over two centuries (Young, 2003).

The problem with using a proportional division occurs in the case of unequal population constituencies. In such a situation it may happen that, when rounded down, for example, the number of seats, some circles in general will be deprived of their representation. This problem is created by the current demographic structure of the European Union. Disparities in the population of Germany and Malta or Luxembourg, lead to the fact that using the method of Webster in the allocation of seats to the European Parliament would deprive two smallest countries of their members in the collegial body. Usage of other known methods does not solve the problem either, since in this case the representation is low. Lack of acceptance of this solution led to the formation of the so-called principle of degressive proportionality.

### Definition of degressive proportionality

Degressive proportionality principle of the allocation of seats in the European Parliament is included in art. Paragraph 9a. 2 of the Treaty of Lisbon. According to this article: "The European Parliament Shall Be Composed of Representatives of the Union's citizens. They shall not exceed seven hundred and fifty in number, plus the President. Representation of citizens shall be **degressively proportional**, with a minimum threshold of six members per Member State. No Member State shall be allocated more than ninety-six seats" (The Treaty of Lisbon, 2010).

However, the Treaty does not contain any guidance on how to understand degressive proportionality. Some explanations can be found in the annex to the draft resolution of the European Parliament (Lamassoure, Severin (2007)). The posted text clarifies the thought contained in the Treaty. The first principle formulated there called the principle of fair division says that a country with a larger number of people cannot get less seats than less populated country. The second, defined as the principle of relative proportionality, concludes that the larger the country, the larger number of voters should be represented by one Member of Parliament.

The formal record of these findings leads to the following definition of degressive proportionality:

**Definition 1.** If  $l_i$ ,  $1 \leq i \leq n$  is a set sequence of decreasing real positive numbers ( $l_1 > l_2 > \dots > l_n$ ), then the sequence of natural numbers  $m_i$ ,  $1 \leq i \leq n$  is degressively proportional with respect to  $l_i$  sequence, if and only if  $m_i$  is non-increasing ( $m_1 \geq m_2 \geq \dots \geq m_n$ ) and  $\frac{m_1}{l_1} < \frac{m_2}{l_2} < \dots < \frac{m_n}{l_n}$ .

In the present definition in sequences  $l_i$  and in quotient sequence  $\frac{m_i}{l_i}$  strong inequalities can be replaced with weak ones. In the first case in practical applications this is of little importance, since it is unlikely that the two constituencies have the same population. In the second, however, allowing weak inequalities results in joining the considerations of proportional division, which seems to be in conflict with the literal wording of the cited principle of relative proportionality. Nevertheless, such definitions can be found in the literature (Florek, 2011). In the quoted work an equivalent formulation of such a definition is also given, it has the form of:

**Definition 1a.** Let  $l_i$ ,  $1 \leq i \leq n$ , be a set non-increasing sequence of real positive numbers, ( $l_1 > l_2 > \dots > l_n$ ), and  $\lceil x \rceil$  denotes rounding the number up to the nearest integer. The sequence of natural numbers  $m_i$ ,  $1 \leq i \leq n$  is degressively proportional with respect to  $l_i$  sequence, if and only if

$$\begin{cases} m_1 = M \\ m_i = \min \left( \left\lceil \frac{m_{i-1}}{l_{i-1}} + a_i \right\rceil, m_{i-1} \right), \quad 2 \leq i \leq n \end{cases}$$

for a certain sequence of  $a_i \geq 0$ ,  $2 \leq i \leq n$ , and for any  $M \in \mathbb{N}$ .

This treatment is interesting as it gives an effective method of constructing degressively proportional sequences.

But this is not the only possible understanding of the ideas contained in the Lisbon Treaty. Degressive proportionality can also be understood as the principle of diminishing of the relative gains of seats. Obviously decreasing with the increase in population. This in turn allows the formulation of degressive proportionality in another form (Cegiełka, 2011):

**Definition 2.** If  $l_i$ ,  $1 \leq i \leq n$  is a set sequence of decreasing real positive numbers ( $l_1 > l_2 > \dots > l_n$ ), then the sequence of natural numbers  $m_i$ ,  $1 \leq i \leq n$  is degressively proportional with respect to  $l_i$  sequence, if and only if  $m_i$  is non-increasing ( $m_1 \geq m_2 \geq \dots \geq m_n$ ), and

$$\frac{m_1}{l_1} < \dots < \frac{\mu_i}{\lambda_i} < \frac{\mu_{i+1}}{\lambda_{i+1}} < \dots < \frac{\mu_n}{\lambda_n}$$

where  $\frac{\mu_i}{\lambda_i} = \begin{cases} \frac{m_{i-1} - m_i}{l_{i-1} - l_i}; & m_{i-1} > m_i \\ \frac{m_{i-j} - m_i}{l_{i-j} - l_i}; & m_{i-j+1} = m_i, \quad j \geq 2 \end{cases}$ .

It is easy to show that the condition formulated in the second definition implies the condition set out in the first definition, but the definitions are not equivalent (Cegiełka, 2011).

There are cases in which the condition of degressive proportionality is fulfilled at the level of quotas allocated to each Member State but is not fulfilled after rounding up to integers. In other words, there are degressively proportional divisions that are not entirely numerical, and they lose this feature after transition to approximations of integers (Misztal, 2011). This lay at the root of constructing yet another definition of degressive proportionality:

**Definition 3.** If  $l_i$ ,  $1 \leq i \leq n$  is a set sequence of decreasing real positive numbers ( $l_1 > l_2 > \dots > l_n$ ), then the sequence of natural numbers  $m_i$ ,  $1 \leq i \leq n$  is degressively proportional with respect to  $l_i$  sequence, if and only if  $m_i$  is non-increasing ( $m_1 \geq m_2 \geq \dots \geq m_n$ ), and  $m_i = [f(l_i)]$  where  $f$  is any real function, increasing and convex, and  $[x]$  denotes rounding the number  $x$  to an integer.

The rounding that occurs in the definition can be understood in any sense, which directs the considerations to the classical proportional methods, in which various ways to bring solutions to the set of integers were also considered.

Further considerations will be conducted under the assumption that degressive proportionality is understood in the sense of Definition 1, that is, the one which is most frequently referred to in official documents (Lamassoure, Severin, 2007).

## Additional recommendations

The quoted article of the Lisbon Treaty in addition to establishing the principle of degressive proportionality also presents further conditions to be fulfilled by the allocation of seats to the European Parliament. The first is to limit the total number of seats to 751, and the next is to set minimum and maximum number of Members of Parliament representing the country at 6 and 96. In the Treaty, these conditions are given in the form of inequalities, thus it is not required to achieve the exact division of numbers specified in the text of the provision, but in the annex to the draft of the European Parliament resolution there can be

found parts explicitly stating that the so-defined boundary conditions are to be met (Lamassoure, Severin, 2007). "The minimum and maximum numbers set by the Treaty must be fully utilized to ensure that the allocation of seats in the European Parliament Reflects as Closely as Possible the range of Populations of the Member States".

This recommendation implicitly emphasizes diversification of the division. In determining the minimum and maximum number of seats it excludes at the same time equal division, granting each country the same number of seats in the European Parliament, and also specifies the minimum diversification of seats. Additional conditions, however, pose a threat of an empty set of solutions. Example of how this situation may occur is given in Table 1.

**Tab. 1 Example of distribution of seats among countries with similar population**

Member States	Population	Number of sets
Greece	11 305 118	21
Belgium	10 839 906	21
Portugal	10 637 713	21
Czech Republic	10 506 813	21
Hungary	10 014 324	21
<b>Total</b>	<b>53 303 874</b>	<b>105</b>

Source: Gimmet, G. et al (2011).

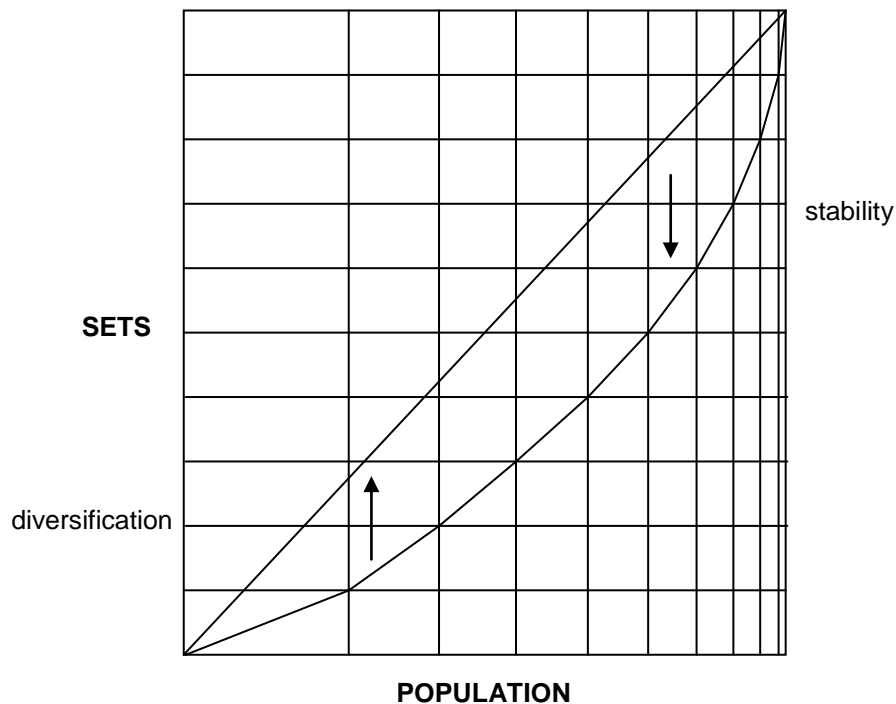
It is easy to notice that in the case of increasing the number of seats to 106, it is impossible to allocate them to those countries in a degressively proportional manner. The first condition of definition 1. requires that it falls to Greece, but then the second condition will not be fulfilled.

The problem of diversification also plays a large role in the implementation of another recommendation. In addition to the previously quoted document, apart from indicating the need to use the full capabilities of the extreme number of seats, it is mentioned also to provide demographic stability of the solutions adopted.

"A clear, comprehensible and transparent system must also be applicable to future changes in the size of the populations of the Member States without substantial new negotiations."

These two recommendations, however, remain contradictory. The most demographically stable is the equal division, which assigns to each member country the same number of seats, it is also the least diversified. On the other hand, proportional allocation, which has the highest diversification is the least resistant to any change in demographics. When illustrating the problem by using empirical Lorenz curve (Łyko, 2010), placing on the axes the accumulated percentage of the population based on the sequence  $l_i$  and accumulated percentage of the seats – sequence  $m_i$ , it is easy to state that degressive divisions place themselves between the lines drawn in Figure 1.

The shift towards the diagonal of the square increases the diversification, whereas the shift towards the lower limit leads to greater stability of the solution.



**Fig. 1 Empirical Lorenz curve of distribution of seats**

Source: own work.

In the issues associated with measuring social inequality Lorenz curve is used as the basis for constructing the meter indicating the scale of disparities in goods or income distribution. Specifically, this measuring tool is the Gini coefficient. It is the doubled area between the Lorenz curve representing a particular division and the diagonal of a square. Given the similarity of the problems, you can use the Gini coefficient as a measure of degressive proportionality of division. A value of zero expresses proportional division. The highest value represents equal division. The value is greater than equality and increases with increasing numbers  $n$ , ie. in the given example, the increase of the number of states, among which the distribution of seats takes place. Gini coefficient for the present assignment is 0,165.

Recommendations of diversification and stability of the demographic shift the burden of searching in two opposite directions. Thus the question about the application of extreme solutions seems to be more interesting. Given the demographic data of 2009 the most diversified is the division of allocating to the Member States 22 different number of seats. Taking into account the considerations forecasts for the years 2015 and 2020, only eight degressively proportional divisions can be constructed that would allocate the number of seats assigned in the Treaty of Lisbon. Their diversification is much smaller. Then it is possible to grant only 7 and 8 different numbers of seats in European Parliament. Moreover, extending the discussion to the year 2024, it appears that such divisions do not exist. It is therefore impossible at present to indicate the composition of the European Parliament, which would for the next three terms satisfy the condition of degressive proportionality.

## Conclusions:

The principle of degressive proportionality specified in the Lisbon Treaty allows for different interpretations of this concept. Lack of clear wording allows to construct many definitions. Additional recommendations contained in the documents regarding the Treaty partly explain this problem, but also bring other demands that cannot be simultaneously

applied. Therefore, divisions are to be sought that offer compromise between the idea of maximum diversification and greater demographic stability.

You may also note that the adopting additional assumptions about the construction of the division, which exactly meets the conditions specified in the Treaty on the total number of deputies and the minimum and maximum representation, clearly narrows the circle of search. Appointing the composition of the European Parliament on the basis of negotiations without any indication of an algorithm used in the process, do not withstand the test of time. Projections of demographic change in European Union countries show that such a solution can survive without violating the principle of degressive proportionality at most two consecutive terms.

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